Math 53, Discussion 107

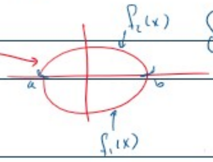
Wednesday, March 18, 2020 9:09 AM

- 15.3 (integral over arbitrary domain
- 15.4 (double integral in polar coordinates)

15.3

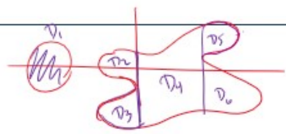
Geometrically $\iint_D f(x,y)\,dA$ is the volume under $z=f(x,y)$ in the region

Ex D



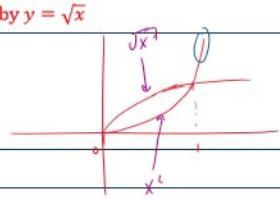
$$\iint_D f(x,y)\,dA = \int_a^b \int_{f_1(x)}^{f_2(x)} f(x,y)\,dy\,dx$$

Ex. D =



15.3 problems:

1. $\int_D (x+y)dA$ when D is bouded by $y=\sqrt{x}$ and $y=x^2$



$$\int_0^1 \int_{x^2}^{\sqrt{x}} (x+y)\,dy\,dx$$

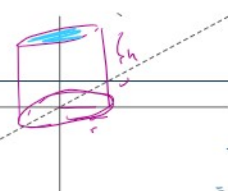
$$xy + \frac{y^2}{2}\Big|_{x^2=y}^{\sqrt{x}=y}$$

$$\int_0^1 x^{3/2} + \frac{x}{2} - x^3 - \frac{x^4}{2}\,dx$$

$$= \frac{2}{5}x^{5/2} + \frac{x^2}{4} - \frac{x^4}{4} - \frac{x^5}{10}\Big|_0^1$$

$$= \frac{2}{5} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{4}} - \frac{1}{10} = \frac{3}{10}$$

1. Set up an integral to evaluate the volume of a cylinder of radius r and height h
2. Set up an integral to evaluate the volume of a sphere of radius r — f won't be constant
3. Find the volume enclosed by the parabolic cylinders $y=1-x^2, y=x^2-1$ and the planes $x+y+z=2, 2x+2y-z+10=0$



if D = circle of radius r.

& $f(x,y)=h$

$= \iint_D f(x,y)\,dA$

= Volume under f in D

= volume of cylinder.

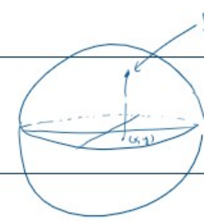
$$\iint_D f\,dA = \iint_D h\,dA$$

$$= \int_{-r}^{r}\int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} h\,dy\,dx$$

$$= h\int_{-r}^{r} y\Big|_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}}$$

$$2h\int_{-r}^{r}\sqrt{r^2-x^2}\,dx$$

$$2hr\int_{-r}^{r}\sqrt{1-\left(\frac{x}{r}\right)^2}\,dx$$



f(x,y) height

Sphere volume

$$= \iint_D f\,dA$$

D = Circle of radius r

f = height of sphere given

$$\int_{-r}^{r}\int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} f(\ldots)\,dA$$

15.4: polar integration

Formula: $\int\int f(r,\theta) r\, dr\, d\theta$

odd

Area of circle $=\pi r^2$

$$= \int_0^r\int_0^{2\pi} 1\, r\,d\theta\,dr = 2\pi\int_0^r r\,dr = 2\pi\frac{r^2}{2}\Big|_0^r$$

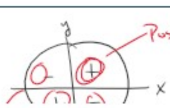
$$= \frac{2\pi r^2}{2} = \pi r^2$$

$s^2+c^2=1$ let $\frac{x}{r}=\sin(\theta)$

$c^2 = 1-s^2$

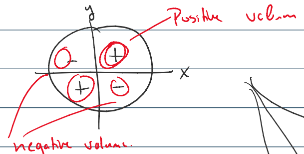
no r => get $2\pi r$

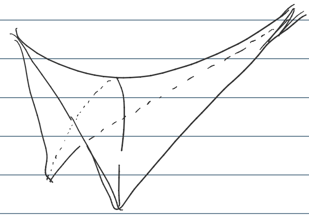
1. Compute $\iint_D xydA$ with D the disk with center at the origin and radius 3



$x = r\cos\theta$ $y = r\sin\theta$

1. Compute $\iint_D xy\,dA$ with D the disk with center at the origin and radius 3





$x = r\cos\theta \qquad y = r\sin\theta$

$$I = \iint_D r\cos\theta\; r\sin\theta\; r\,dr\,d\theta = \int_0^{2\pi}\int_0^{3} r^3 \sin\theta\cos\theta\,dr\,d\theta$$

$\frac{\sin(2\theta)}{2}$

$$= \int_0^{2\pi} \sin\theta\cos\theta \left.\frac{r^4}{4}\right|_{r=0}^{r=3} d\theta = \frac{81}{4}\int_0^{2\pi} \sin\theta\cos\theta\,d\theta = \frac{81}{4}\left.\frac{\sin^2\theta}{2}\right|_0^{2\pi} = 0$$

$\frac{d}{d\theta}\left(\frac{\sin^2(\theta)}{2}\right)$

(1) Compute the area of one loop of the rose $r = \cos 3\theta$

2. Compute the area of a sphere of radius a

3. Fun: learn the trick how to compute $\int_{-\infty}^{\infty} e^{-x^2}\,dx$ (area under a Gaussian)

$\int_0^r r\,dr$

had formula

$$\boxed{\int_{\theta_1}^{\theta_2} \frac{r^2}{2}\,dr}$$